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NBS TECHNICAL NOTE 595

Effect of Ceramic Spectral Emissivity Variations on the Computed Luminous Emissivity of the NBS Standard of Light

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NBS TECHNICAL NOTE 595

ISSUED SEPTEMBER 1971

Nat. Bur. Stand. (U.S.), Tech. Note 595, 58 pages (Sept. 1971)
CODEN: NBTNA

Effect of Ceramic Spectral Emissivity Variations on the Computed Luminous Emissivity of the NBS Standard of Light

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Effect of Ceramic Spectral Emissivity Variations on the Computed Luminous Emissivity of the NBS Standard of Light

William B. Fussell

A simplified model of the National Bureau of Standards (NBS) 1970 design for the standard of light (platinum point blackbody) is introduced. This model is used to calculate the apparent luminous emissivity of the base of the sighttube of the standard of light, and of the wall near the base, by two different methods. The first method includes the effect of variations with wavelength in the spectral emissivity of the ceramic composing the sighttube; the second method does not include the effect of ceramic spectral emissivity variations. The results of the two methods are compared and their difference (about $6x10^{-6}$) is found to be negligible for the NBS 1970 design when the sighttube is made of alumina, compared with other uncertainties and the precision of measurement. A formula is derived for estimating the apparent luminous emissivity of the standard of light, given: a., the length-to-diameter ratio of the sighttube; b., the half-angle of the conical base; c., the wall thickness of the sighttube; d., the thermal conductivity of the sighttube ceramic; e., the average thermal and luminous emissivities of the sighttube ceramic.

Keywords: Ceramic; light; photometry; spectral emissivity; standard.

1. Introduction

The standard of light is defined to be a blackbody at the freezing point of platinum (this is a secondary reference point in the International Practical Temperature Scale of 1968 [1][1]; its value is 2045 K); the luminance of this blackbody is defined to be 60 candelas per square centimeter. The physical approximation to this blackbody is also known as the standard of light.

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[1] Figures in square brackets indicate the literature references at the end of this paper.

In connection with setting up the standard of light at the National Bureau of Standards (NBS), it is important to calculate, as accurately as possible, the apparent luminous emissivity[1] of the standard, given its geometry and the material of which its sighttube is composed (see fig. 1). C. L. Sanders of the National Research Council (NRC; Canada) has developed a computer program [2] for this purpose that: a., computes the temperature distribution inside the sighttube of the standard of light, given the geometry and the thermal conductivity and average thermal emissivity of the ceramic composing the sighttube; b., then uses the temperature distribution of a. to compute the corresponding distribution of the apparent spectral emissivity inside the sighttube at the appropriate Crova wavelength (about 0.578 micrometer [3]), given the average luminous emissivity of the ceramic. The apparent spectral emissivity at the Crova wavelength of an area inside the sighttube is assumed to be equal to the apparent luminous emissivity of this area. This is a valid assumption if the spectral emissivity of the ceramic composing the sighttube is constant with wavelength; however, if the spectral emissivity of the ceramic is not constant with wavelength (the spectral emissivity of alumina, for example, varies widely with wavelength), then the apparent spectral emissivity at the Crova wavelength will differ from the apparent luminous emissivity.

The goal of this paper is to calculate approximately this difference between the apparent spectral emissivity at the Crova wavelength and the apparent luminous emissivity, for the base and the adjacent wall of the sighttube of the present design for the National Bureau of Standards (NBS) standard of light, composed of alumina. (This will also be referred to as the NBS 1970 configuration.)

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[1] Specialized terms and quantities are defined in the list of terminology at the end of this introduction.

1.1. Assumptions of the Sanders-NRC Computer Program

a. The ceramic composing the sighttube of the standard of light emits and reflects diffusely[1].

b. The portion of the outside wall of the sighttube immersed in freezing platinum is maintained at the freezing point exactly (that is, temperature gradients in the platinum ingot are ignored).

c. In the computation of the temperature distribution in the interior of the sighttube, the variation with wavelength in the spectral emissivity of the ceramic may be approximated by substituting, in the spectral radiation balance equations for the base and the wall zones (see fig. 1), the average thermal emissivity of the ceramic for the spectral emissivity.

d. In the computation of the distribution of the apparent luminous emissivity in the interior of the sighttube, the variation with wavelength in the spectral emissivity of the ceramic in the visible may be approximated by substituting, in the spectral radiation balance equations for the base and the wall zones (see fig. 1) at the Crova wavelength, the average luminous emissivity of the ceramic for the spectral emissivity.

---

[1] In this paper, the terms "diffuse" and "diffusely" refer to reflection or emission that follows Lambert's cosine law [4].

1.2. Calculations Required to Compute the Effect of Ceramic Spectral Emissivity Variations with Wavelength on the Accuracy of the Apparent Luminous Emissivities Produced by the Sanders-NRC Computer Program

To calculate approximately the effect of the variation with wavelength in the spectral emissivity of the ceramic, on the accuracy of the apparent luminous emissivity distribution in the interior of the sighttube produced by the Sanders-NRC computer program, it is necessary to proceed as follows:

a. Calculate the effect of ceramic spectral emissivity variations with wavelength on the temperature distribution inside the sighttube computed by the Sanders-NRC program.

b. Calculate the effect of ceramic spectral emissivity variations with wavelength, in the visible, on the apparent luminous emissivity distribution inside the sighttube computed by the Sanders-NRC program.

c. Combine effects a. and b. to calculate the total effect, of ceramic spectral emissivity variations with wavelength, on the apparent luminous emissivity distribution in the interior of the sighttube computed by the Sanders-NRC program.

1.3. Simplified Model of the NBS 1970 Design for the Standard of Light

To enable these approximate calculations to be done efficiently, a simplified model of the NBS 1970 design for the standard of light is adopted. The model is simplified as follows:

a. The length-to-internal diameter ratio of the sighttube is taken to be 20 (the actual value is 19.0).

b. The half-angle of the conical base of the sighttube is taken as $\sin^{-1}(0.2)$ (11.5°; the actual value is 9.5°).

c. The interior of the sighttube is divided into 20 wall zones, each of length equal to the internal diameter, plus the conical base (see fig. 1).

d. In the spectral radiation balance equations for the base and the adjacent wall zone (wall zone 1, in fig. 1), the remaining wall zones are treated as perfect black radiators. This assumption is justified in the mathematical analysis section of this paper; it is based on the rapid falloff of the viewfactors between zones with increasing separation (see the viewfactor matrix given in table 1). For this reason, the spectral deficiencies (1 minus the apparent spectral emissivities; see the list of terminology at the end of this introduction) of distant zones - if the deficiencies are not too large - have little effect on the spectral radiation balance equation of a zone.

1.4. Scheme of the Calculation of the Effect of Ceramic Spectral Emissivity Variations with Wavelength on the Accuracy of the Apparent Luminous Emissivities Produced by the Sanders-NRC Computer Program for the Simplified Model of the NBS 1970 Design for the Standard of Light

a. The spectral radiation balance equations for the base and the adjacent wall zone (wall zone 1) are set up and linearized by the assumption (justified in the mathematical analysis section of this paper) that their normalized temperature drops (that is, the temperature drops in the sighttube wall, divided by the freezing point of platinum) are much less than 1.

b. Two simultaneous linear equations for the spectral deficiencies of the base and wall zone 1 are then derived from a. by the use of the modified nearest-neighbor model outlined in 1.3.d, above.

c. The pair of simultaneous linear equations, b., are solved for the spectral deficiencies of the base and wall zone 1.

d. The power balance equations are set up for the base and wall zone 1, and also linearized. These equations involve the spectral deficiencies of the base and wall zone 1, computed in c., which appear in the integrand of an integral with respect to the wavelength from (nominally) zero to infinity. The power balance equations also involve the normalized base and wall-zone 1 temperature drops.

e. The power balance equations, d., are now considered as simultaneous linear equations in the normalized base and wall-zone 1 temperature drops, and are solved for these quantities; the equations are simplified by neglecting terms involving the square of G (G is 1/4 the ratio of the hemispherical radiative conductance to the thermal conductance through the wall of the sighttube, and is equal to 0.0619 for the NBS 1970 design for the standard of light if the sighttube is composed of alumina) and retaining only terms involving the first power of G (this approximation is justified in the mathematical analysis section).

The solutions for the normalized base and wall-zone 1 temperature drops then involve integrals, with respect to the wavelength from (nominally) zero to infinity, of integrands that are rational functions of the ceramic spectral emissivity with coefficients that involve the viewfactors.

f. Numerical values for the base and wall-zone 1 temperature drops are obtained by numerical integration of these integrals; to permit evaluation of the advantage of the conical base configuration over the flat base configuration, the normalized base and wall-zone 1 temperature drops are also calculated for the flat base. The results are given in table 2 (cols. 2 and 5).

g. The values obtained in f. for the base and wall-zone 1 temperature drops include the effect of ceramic spectral emissivity variations with wavelength; to approximate the values obtained by the Sanders-NRC computer program, which do not include the effect of ceramic spectral emissivity variations with wavelength, the ceramic spectral emissivity, in the solutions, e., for the normalized base and wall-zone 1 temperature drops, is replaced by the average thermal emissivity of the ceramic. Numerical values are then calculated for the normalized base and wall-zone 1 temperature drops by evaluating the integrals in their solutions. These results are also given in table 2 (cols. 4 and 7).

h. The difference between the normalized temperature drops calculated in f., and those calculated in g., is approximately the magnitude of the error in the corresponding quantities, derived from the base and wall-zone 1 temperatures calculated by the Sanders-NRC computer program, due to ceramic spectral emissivity variations with wavelength.

i. Return now to the spectral radiation balance equations, b., in terms of the spectral deficiencies of the base and wall zone 1. The simultaneous solution of the pair of equations, b., yields solutions for the spectral deficiencies of the base and wall zone 1 in terms of:

(1) the spectral emissivity of the ceramic at the wavelength under consideration;

(2) the normalized base and wall-zone 1 temperature drops;

(3) the viewfactors for the aperture as viewed from the base and wall zone 1;

(4) the viewfactor for the base as viewed from wall zone 1, and for wall zone 1 as viewed from the base;

(5) the self-viewfactors for the base and wall zone 1;

(6) the partial derivative of the logarithm of the Planck blackbody spectral radiance function, at the wavelength under consideration and at the freezing point of platinum, with respect to the logarithm of the temperature.

The quantities (1)-(6) are known, since the normalized base and wall-zone 1 temperature drops have been calculated in f. and g., above.

To include the effect of variations in the ceramic spectral emissivity with wavelength, in computing the apparent luminous emissivity of the base and wall zone 1, insert the normalized base and wall-zone 1 temperature drops - derived in f., above - which include the effect of ceramic spectral emissivity variations, into the equations derived in c., above, for the base and wall-zone 1 spectral deficiencies. Multiply the resulting equations by:

(1) the spectral luminous efficiency function for photopic vision [5];

(2) the Planck blackbody spectral radiance function for the wavelength under consideration and the freezing point of platinum. The resulting equations are then integrated with respect to the wavelength over the visible spectrum (0.38 - 0.78 micrometer [6]), and divided by a normalizing constant, to obtain the luminous deficiencies of the base and wall zone 1, with the effect of ceramic spectral emissivity variations with wavelength included. These quantities are given in table 3 (cols. 2 and 5).

j. To compute the luminous deficiencies of the base and wall zone 1 without taking account of ceramic spectral emissivity variations with wavelength, return to the solutions, c., for the spectral deficiencies of the base and wall zone 1. Insert the normalized temperature drops of the base and wall zone 1 - derived in g., above - which do not include the effect of ceramic spectral emissivity variations with wavelength, into the equations derived in c. for the spectral deficiencies of the base and wall zone 1. Then evaluate these equations at the Crova wavelength by replacing the ceramic spectral emissivity by the average luminous emissivity of the ceramic. The resulting values for the luminous deficiencies of the base and wall zone 1 are also shown in table 3 (cols. 4 and 7), and are assumed to approximate the corresponding values obtained by the Sanders-NRC computer program, since the computational techniques are similar.

1.5. Outline of the Sanders-NRC Computer Program

For a sighttube divided into N+1 zones (N wall zones plus the base), the Sanders-NRC computer program solves simultaneously - by iteration - the system of N+1 power balance equations plus N+1 apparent thermal radiation balance equations for:

a. The N+1 interior temperatures of the zones.

b. The N+1 apparent thermal emissivities of the zones.

The N+1 power balance equations are not linearized in the Sanders-NRC program, and are solved by Newton-Raphson iteration. After completing steps a. and b., above, the Sanders-NRC program uses the N+1 interior temperatures to solve simultaneously - again by iteration - the system of N+1 apparent luminous radiation balance equations at the Crova wavelength for:

c. The N+1 apparent luminous emissivities of the zones.

The flow charts shown in figures 2, 3, and 4, clarify the distinction between the luminous deficiencies calculated in 1.4.i, those calculated in 1.4.j, and those calculated by the Sanders-NRC computer program.

1.6. Summary of Approximations Used in Computing the Effect of Ceramic Spectral Emissivity Variations with Wavelength on the Apparent Luminous Emissivities Produced by the Sanders-NRC Computer Program for the Simplified Model of the NBS 1970 Design for the Standard of Light

a. All zones are treated as perfect black radiators at the freezing point of platinum except the base and wall zone 1 (modified nearest-neighbor approximation).

b. The spectral radiation balance equations and the power balance equations for the base and wall zone 1 are linearized with respect to the spectral deficiencies and the normalized temperature drops (that is, these quantities are assumed to be small).

c. The normalized temperature drop of the base is neglected in computing the normalized temperature drop of wall zone 1, and vice versa (that is, only the first power of G is retained in the power balance equations).

1.7. Fundamental Assumption Relating the Apparent Luminous Emissivities Produced by the Sanders-NRC Computer Program and the Apparent Luminous Emissivities Computed from the Modified Nearest-Neighbor Approximation

The difference between the luminous deficiencies calculated in 1.4.i, and those calculated in 1.4.j, is assumed to be approximately the magnitude of the error in the luminous deficiencies calculated by the Sanders-NRC computer program, due to neglect of the ceramic spectral emissivity variations with wavelength. This assumption is justified by an approximate calculation of the errors involved in the modified nearest-neighbor approximation, used to compute the luminous deficiencies of 1.4.i and 1.4.j. An evaluation of the most important terms omitted from the modified nearest-neighbor approximation shows that these terms do not significantly affect the results obtained with this approximation.

Table 3 shows that the fractional error due to neglect of ceramic spectral emissivity variations with wavelength is approximately 2.5% for the luminous deficiency of the base, for the NBS 1970 configuration made of alumina; the corresponding fractional error for wall zone 1 is about 4.4%.

Since the quantities in table 3 (except for the fractional errors) are divided by the appropriate viewfactor for the aperture, as viewed from the base (see table 1 for the viewfactor matrix), the actual luminous deficiencies are quite small; it is found that the luminous deficiencies for the base and wall zone 1, computed by method 1.4.i, which takes ceramic spectral emissivity variations with wavelength into account, are $2.44\mathrm{x}10^{-4}$ and $0.878\mathrm{x}10^{-4}$, respectively; the corresponding values for method 1.4.j, which simulates the Sanders-NRC program and does not take ceramic spectral emissivity variations into account, are $2.38\mathrm{x}10^{-4}$ and $0.839\mathrm{x}10^{-4}$, respectively.

Thus the estimated absolute errors produced by the Sanders-NRC computer program, due to neglect of ceramic spectral emissivity variations with wavelength, in the apparent luminous emissivities of the base and wall zone 1 of the NBS 1970 design for the standard of light made of alumina, are on the order of $6\mathrm{x}10^{-6}$ for the base and $4\mathrm{x}10^{-6}$ for wall zone 1. These uncertainties are insignificant in comparison with the much larger uncertainties due to other causes.

1.8. Terminology

A quantity with the subscript "j" refers to the interior surface of the j-th zone of the sighttube of the NBS 1970 design for the standard of light (see fig. 1). "j" runs from "B" (for the base of the sighttube), through "1" (for the first wall zone), to "N" (for the last wall zone), and finally to "A" (for the aperture of the sighttube).

The reference temperature for apparent emissivities and for deficiencies is the freezing point of platinum, 2045 K [1].

All surfaces are assumed to emit and reflect diffusely.

A single bar over a radiative quantity denotes the "thermal average" of the spectrum of the quantity; the thermal average is defined as the average value of the spectrum of the quantity, weighted by the Planck blackbody spectral radiance function at the freezing point of platinum, over the wavelength range from (nominally) zero to infinity.

A double bar over a radiative quantity denotes the "luminous average" of the spectrum of the quantity; the luminous average is defined as the average value of the spectrum of the quantity, weighted by the product of the Planck blackbody spectral radiance function at the freezing point of platinum and the spectral luminous efficiency function for photopic vision, over the visible spectrum.

The single apostrophe after a quantity denotes a "normalized" quantity; that is, the value of the quantity lies between zero and 1.

The double apostrophe after a quantity denotes a normalized quantity that has been divided by the viewfactor of the aperture of the sighttube (for the NBS 1970 design) as seen from the base (this viewfactor is defined below). (The one exception to this rule is $J''_\lambda(\lambda,T_F)$; see below.)

The subscript "0" preceding a quantity denotes the "zero-order" approximation to the quantity (this is explained in the mathematical analysis section of this paper that follows the introduction).

The term "deficiency" refers to 1 minus the corresponding "emissivity".

R_1 (Outside Radius of the Sighttube): This is about 0.3 cm for the NBS 1970 configuration.

R_2 (Inside Radius of the Sighttube): This is about 0.2 cm for the NBS 1970 configuration.

L (Length of the Cylindrical Portion of the Sighttube): This is about 7.54 cm for the NBS 1970 configuration.

t (Effective Thickness of the Sighttube Wall): This is about 0.081 cm for the NBS 1970 configuration; t is defined mathematically as,

$$t = R_2 \ln(R_1/R_2).$$

θ (Half-Angle of the Conical Base of the Sighttube): This is about 9.5° for the NBS 1970 configuration, but the approximate value of $\sin^{-1}(0.2)$ ($11^\circ 32'$) is used for the calculations in this paper.

$\bar{L}$ (Length-to-Inside Radius Ratio of the Sighttube): This is about 38.1 for the NBS 1970 configuration, but the approximate value of 40 is used for the calculations in this paper; $\bar{L}$ is defined mathematically as

$$\bar{L} = L/R_2.$$

F_{jk} (Diffuse Viewfactor of the j-th Zone as Viewed from the k-th Zone): The diffuse viewfactor of the j-th zone as viewed from the k-th zone is the fraction of the total radiation from the k-th zone (assuming it radiates diffusely with uniform radiance) intercepted by the j-th zone. (The viewfactor matrix for the NBS 1970 configuration is shown in table 1; this matrix is derived from the basic disc-disc viewfactor formula given in ref. 7.)

F_{AB}^{0} (Aperture-Base Viewfactor for the Flat Base Configuration): This is defined mathematically by the equation,

$$F_{AB} = F_{AB}^{0} \sin\theta.$$

k (Thermal Conductivity of the Sighttube Ceramic): This is estimated to be 0.0635 W cm^{-1} K^{-1} at 2045 K for the high-purity, zero-porosity alumina used in the NBS 1970 design for the standard of light [8,9].

λ (Wavelength).

T_F (Freezing Point of Platinum): The value of this secondary reference point in the International Practical Temperature Scale of 1968 is 2045 K [1].

T_j (Interior Temperature of the j-th Zone): The average interior temperature of the j-th zone of the sighttube of the NBS 1970 design for the standard of light (see fig. 1).

T_j' (Normalized Interior Temperature of the j-th Zone): This is defined mathematically as

$$T_j' = T_j/T_F.$$

δT_j (Temperature Drop of the j-th Zone): This is defined mathematically as

$$\delta T_j = T_F - T_j.$$

$\delta T_j'$ (Normalized Temperature Drop of the j-th Zone): This is defined mathematically as

$$\delta T_j' = 1 - T_j'.$$

$J_\lambda(\lambda,T_F)$ (Planck Blackbody Spectral Exitance Function at Wavelength λ and Temperature T_F): This is defined mathematically as

$$J_\lambda(\lambda,T_F) = c_1\lambda^{-5}(\exp(c_2/\lambda T_F)-1)^{-1}.$$

c_1 (First Radiation Constant): The most recent value is $3.7418\text{x}10^{-16}$ W m^2 [10].

c_2 (Second Radiation Constant): The International Practical Temperature Scale of 1968 defines c_2 to be $1.4388\text{x}10^{-2}$ m K [1].

$J'_\lambda(\lambda,T_F)$ (Normalized Planck Blackbody Spectral Exitance Function at Wavelength λ and Temperature T_F): This is defined mathematically as

$$J'_\lambda(\lambda,T_F) = J_\lambda(\lambda,T_F)/\sigma T_F^4.$$

σ (Stefan-Boltzmann Constant): The most recent value is $5.6696\text{x}10^{-8}$ W m^{-2}K^{-4} [10].

(Note that σ, c_1, and c_2, are related by the equation,

$$\sigma = (\pi^4/15)c_1c_2^{-4};$$

note also that

$$\int_0^\infty J'_\lambda(\lambda,T_F)d\lambda = 1.)$$

$N(\lambda,T_F)$ (Exponent of Temperature Dependence of the Planck Blackbody Spectral Exitance Function at Wavelength λ and Temperature T_F): This is defined mathematically as

$$N(\lambda,T_F) = d[\ln J_\lambda(\lambda,T_F)]/d[\ln T_F].$$

e (Emissivity): The emissivity of an isothermal radiating surface is the ratio of the power radiated (but not reflected) by the surface to a specified receiving surface, to the corresponding power radiated by a perfectly black surface at the same temperature and having the same configuration.

$e(\lambda)$ (Spectral Emissivity): The spectral emissivity of an isothermal radiating surface at wavelength λ is the ratio of the power radiated (but not reflected) by the surface in a small wavelength interval to a specified receiving surface, to the corresponding power radiated by a perfectly black surface at the same temperature and having the same configuration.

$r(\lambda)$ (Spectral Reflectivity): This is defined mathematically as

$$r(\lambda) = 1 - e(\lambda).$$

$\bar{e}$ (Average Thermal Emissivity): This is defined mathematically as

$$\bar{e} = \int_0^\infty e(\lambda) J_\lambda'(\lambda, T_F) d\lambda;$$

$\bar{e}$ is found to be 0.223 for the ceramic (alumina [11]) used in the NBS 1970 design for the standard of light.

$\bar{r}$ (Average Thermal Reflectivity): This is defined mathematically as

$$\bar{r} = 1 - \bar{e};$$

thus $\bar{r}$ is 0.777 for the ceramic (alumina) used in the NBS 1970 design for the standard of light.

W_j' (Apparent Emissivity of the j-th Zone): The apparent emissivity of a radiating surface (not necessarily isothermal) is the ratio of the total power radiated and reflected (from the surroundings) by the surface to a specified receiving surface, to the corresponding power radiated by a perfectly black surface having the same configuration and isothermal at the given reference temperature (T_F, for the standard of light).

$W'_j(\lambda)$ (Apparent Spectral Emissivity of the j-th Zone): The apparent spectral emissivity of a radiating surface (not necessarily isothermal) at wavelength λ is the ratio of the total power radiated and reflected (from the surroundings) by the surface in a small wavelength interval to a specified receiving surface, to the corresponding power radiated by a perfectly black surface having the same configuration and isothermal at the given reference temperature (T_F, for the standard of light).

$\delta'_j(\lambda)$ (Spectral Deficiency of the j-th Zone): The spectral deficiency of a radiating surface (not necessarily isothermal) at wavelength λ is 1 minus the apparent spectral emissivity of the surface at this wavelength; that is,

$$\delta'_j(\lambda) = 1 - W'_j(\lambda).$$

$\bar{W}'_j$ (Apparent Thermal Emissivity of the j-th Zone): This is defined mathematically as

$$\bar{W}'_j = \int_0^\infty W'(\lambda) J'_\lambda(\lambda, T_F)\, d\lambda.$$

$\bar{\delta}'_j$ (Thermal Deficiency of the j-th Zone): This is defined mathematically as

$$\bar{\delta}'_j = 1 - \bar{W}'_j.$$

$\bar{N}$ (Average Thermal Temperature Exponent): This is defined mathematically as

$$\bar{N} = \int_0^\infty N(\lambda, T_F) J'_\lambda(\lambda, T_F)\, d\lambda = 4.$$

$\bar{e}'$ (Temperature-Exponent Weighted Average Thermal Emissivity): This is defined mathematically as

$$\bar{e}' = \int_0^\infty e(\lambda)N(\lambda,T_F)J_\lambda(\lambda,T_F)d\lambda/(4\sigma T_F^4);$$

$\bar{e}'$ is found to be 0.204 for the ceramic (alumina [11]) used in the NBS 1970 design for the standard of light.

$\bar{e}_2'^2$ (Temperature-Exponent Weighted, Average Thermal Square of the Emissivity): This is defined mathematically as

$$\bar{e}_2'^2 = \int_0^\infty e^2(\lambda)N(\lambda,T_F)J_\lambda(\lambda,T_F)d\lambda/(4\sigma T_F^4);$$

$\bar{e}_2'$ is found to be 0.230 for the ceramic (alumina [11]) used in the NBS 1970 design for the standard of light.

G (1/4 the Ratio of the Hemispherical Radiative Conductance to the Thermal Conductance through the Wall of the Sighttube): This is defined mathematically as the dimensionless quantity,

$$G = \sigma T_F^3 t/k;$$

G is found to be 0.0619 for the ceramic (alumina) and the wall thickness used in the NBS 1970 design for the standard of light.

${}_0\delta_j'(\lambda)$ (Zero-Order Spectral Deficiency of the j-th Zone): This is defined mathematically (see the mathematical analysis section of this paper) as

$${}_0\delta_j'(\lambda) = F_{Aj}[r(\lambda) + e(\lambda)N(\lambda,T_F)G\bar{e}(1-\bar{r}F_{jj})^{-1}][1 - r(\lambda)F_{jj}]^{-1}.$$

$E_j(\lambda)$ (Approximate Fractional Error of the Zero-Order Spectral Deficiency of the j-th Zone, Compared with the Corresponding Nearest-Neighbor Spectral Deficiency): This is defined mathematically (see the mathematical analysis section of this paper) as

$$E_j(\lambda) = [r^2(\lambda)(F_{A,j-1}F_{j-1,j} + F_{A,j+1}F_{j+1,j})]$$

$$\times [F_{Aj}(r(\lambda) + e(\lambda)N(\lambda,T_F)G\bar{e}(1-\bar{r}F_{jj})^{-1}]^{-1}.$$

${}_0\bar{\delta}_j'$ (Zero-Order Thermal Deficiency of the j-th Zone): This is defined mathematically (see the mathematical analysis section of this paper) as

$${}_0\bar{\delta}_j' = F_{Aj}[\bar{r} + 4G\bar{e}^2(1-\bar{r}F_{jj})^{-1}][1 - \bar{r}F_{jj}]^{-1}.$$

$\bar{E}_j$ (Approximate Fractional Error of the Zero-Order Thermal Deficiency of the j-th Zone, Compared with the Corresponding Nearest-Neighbor Thermal Deficiency): This is defined mathematically (see the mathematical analysis section of this paper) as

$$\bar{E}_j = [\bar{r}^2(F_{A,j-1}F_{j-1,j} + F_{A,j+1}F_{j+1,j})][F_{Aj}(\bar{r} + 4G\bar{e}^2(1-\bar{r}F_{jj})^{-1}]^{-1}.$$

${}_0\delta T_j'$ (Zero-Order Normalized Temperature Drop for the j-th Zone): This is defined mathematically (see the mathematical analysis section of this paper) as

$${}_0\delta T_j' = F_{Aj}G\bar{e}(1 - \bar{r}F_{jj})^{-1}.$$

$E_j({}_0\delta T_j')$ (Approximate Fractional Error of the Zero-Order Normalized Temperature Drop of the j-th Zone, Compared with the Corresponding Nearest-Neighbor Normalized Temperature Drop): This is defined mathematically (see the mathematical analysis section of this paper) as

$$E_j({}_0\delta T'_j) = [\bar{r}/F_{Aj}][F_{A,j-1}F_{j-1,j}(1-\bar{r}F_{j-1,j-1})^{-1} + F_{A,j+1}F_{j+1,j}(1-\bar{r}F_{j+1,j+1})^{-1}][1 - \bar{r}F_{jj}].$$

$\bar{\delta}''_j$ (Thermal Deficiency of the j-th Zone, Divided by the Aperture-Base Viewfactor): This is defined mathematically as

$$\bar{\delta}''_j = \bar{\delta}'_j/F_{AB}.$$

$\delta T''_j$ (Normalized Temperature Drop for the j-th Zone,Divided by the Aperture-Base Viewfactor): This is defined mathematically as

$$\delta T''_j = \delta T'_j/F_{AB}.$$

$\delta T''_j(\bar{e})$ (Normalized Temperature Drop for the j-th Zone, Divided by the Aperture-Base Viewfactor, Computed from $\bar{e}$): $\delta T''_B(\bar{e})$ and $\delta T''_1(\bar{e})$ are computed from the modified nearest-neighbor power balance equations (see the mathematical analysis section of this paper) with the spectral emissivity of the ceramic, $e(\lambda)$, replaced by the average thermal emissivity, $\bar{e}$. $\delta T''_B(\bar{e})$ and $\delta T''_1(\bar{e})$ are assumed to approximate the corresponding temperature drops computed by the Sanders-NRC computer program, since the computational procedures are similar; these quantities do _not_ take into account variations in the spectral emissivity of the ceramic with wavelength.

$\delta T''_j(e(\lambda))$ (Normalized Temperature Drop for the j-th Zone, Divided by the Aperture-Base Viewfactor, Computed from $e(\lambda)$): $\delta T''_B(e(\lambda))$ and $\delta T''_1(e(\lambda))$ are computed from the modified nearest-neighbor power balance equations, obtained by numerical integration with respect to the wavelength from (nominally) zero to infinity of the appropriate rational functions of the ceramic spectral emissivity (see the mathematical analysis section of this paper); these quantities do take into account variations in the spectral emissivity of the ceramic with wavelength.

$V(\lambda)$ (Spectral Luminous Efficiency Function for Photopic Vision): The numerical values of $V(\lambda)$ are given in reference 6.

$J''_\lambda(\lambda,T_F)$ (Planck Blackbody Spectral Exitance Function at Wavelength λ and Temperature T_F, Normalized for "Light Watts" per Square Centimeter): This is defined mathematically as

$$J''_\lambda(\lambda,T_F) = 3.5704c_1\lambda^{-5}(\exp(c_2/\lambda T_F)-1)^{-1}.$$

(Note that $J''_\lambda(\lambda,T_F)$ is an exception to the rules of terminology given at the beginning of this section on terminology; also note that

$$\int_0^\infty V(\lambda)J''_\lambda(\lambda,T_F)d\lambda = 1,$$

since

$$3.5704 = K_m/60\pi.)$$

K_m (Maximum Luminous Efficacy for Photopic Vision): The currently accepted value is 673 lm W^{-1} [12].

$\bar{\bar{N}}$ (Average Luminous Temperature Exponent): This is defined mathematically as

$$\bar{\bar{N}} = \int_0^\infty N(\lambda,T_F)V(\lambda)J''_\lambda(\lambda,T_F)d\lambda = 12.15.$$

λ_C (Crova Wavelength at Temperature T_F): This is defined mathematically by the equation,

$$N(\lambda_C,T_F) = \bar{\bar{N}};$$

λ_C is found to be 0.578 micrometer [3].

$\bar{\bar{e}}$ (Average Luminous Emissivity): This is defined mathematically as

$$\bar{\bar{e}} = \int_0^\infty e(\lambda)V(\lambda)J''_\lambda(\lambda,T_F)d\lambda;$$

$\bar{\bar{e}}$ is found to be 0.344 for the ceramic (alumina [13]) used in the NBS 1970 design for the standard of light.

$\bar{\bar{r}}$ (Average Luminous Reflectivity): This is defined mathematically as

$$\bar{\bar{r}} = 1 - \bar{\bar{e}}.$$

$\bar{\bar{W}}'_j$ (Apparent Luminous Emissivity of the j-th Zone): This is defined mathematically as

$$\bar{\bar{W}}'_j = \int_0^\infty W'_j(\lambda)V(\lambda)J''_\lambda(\lambda,T_F)d\lambda.$$

$\bar{\bar{\delta}}'_j$ (Luminous Deficiency of the j-th Zone): This is defined mathematically as

$$\bar{\bar{\delta}}'_j = 1 - \bar{\bar{W}}'_j.$$

${}_0\bar{\bar{\delta}}'_j$ (Zero-Order Luminous Deficiency of the j-th Zone): This is defined mathematically (see the mathematical analysis section of this paper) as

$${}_0\bar{\bar{\delta}}'_j = F_{Aj}[\bar{\bar{r}} + \bar{\bar{N}}\bar{\bar{e}}G\bar{e}(1-\bar{r}F_{jj})^{-1}][1 - \bar{\bar{r}}F_{jj}]^{-1}.$$

$\bar{\bar{E}}_j$ (Approximate Fractional Error of the Zero-Order Luminous Deficiency of the j-th Zone, Compared with the Corresponding Nearest-Neighbor Luminous Deficiency): This is defined mathematically (see the mathematical analysis section of this paper) as

$$\bar{\bar{E}}_j = \bar{\bar{r}}^2[F_{A,j-1}F_{j-1,j} + F_{A,j+1}F_{j+1,j}][F_{Aj}(\bar{\bar{r}} + \bar{\bar{N}}\bar{\bar{e}}G\bar{e}(1-\bar{r}F_{jj})^{-1})]^{-1}.$$

$\bar{\bar{\delta}}''_j$ (Luminous Deficiency of the j-th Zone, Divided by the Aperture-Base Viewfactor): This is defined mathematically as

$$\bar{\bar{\delta}}''_j = \bar{\bar{\delta}}'_j/F_{AB}.$$

${}_0\bar{\bar{\delta}}'_B(\theta)$ (Zero-Order Luminous Deficiency of the Base, as a Function of the Half-Angle, θ, of the Conical Base of the Sighttube): This is defined mathematically (see the concluding section of this paper) as

$${}_0\bar{\bar{\delta}}'_B(\theta) = F^0_{AB}\sin\theta[\bar{\bar{r}} + \bar{\bar{N}}\bar{\bar{e}}G\bar{e}(\bar{e}+\bar{r}\sin\theta)^{-1}][\bar{\bar{e}} + \bar{\bar{r}}\sin\theta]^{-1}.$$

$\bar{\bar{\delta}}''_j(\bar{\bar{e}})$ (Luminous Deficiency of the j-th Zone, Divided by the Aperture-Base Viewfactor, Computed from $\bar{\bar{e}}$): $\bar{\bar{\delta}}''_B(\bar{\bar{e}})$ and $\bar{\bar{\delta}}''_1(\bar{\bar{e}})$ are computed from the modified nearest-neighbor spectral radiation balance equations (see the mathematical analysis section of this paper) at the Crova wavelength, with the spectral emissivity of the ceramic, e(λ), replaced by the average luminous emissivity, $\bar{\bar{e}}$. $\bar{\bar{\delta}}''_B(\bar{\bar{e}})$ and $\bar{\bar{\delta}}''_1(\bar{\bar{e}})$ are assumed to approximate the corresponding quantities derived from the Sanders-NRC computer program, since the computational procedures are similar; these quantities do not take into account variations in the spectral emissivity of the ceramic with wavelength.

$\bar{\bar{\delta}}''_j(e(\lambda))$ (Luminous Deficiency of the j-th Zone, Divided by the Aperture-Base Viewfactor, Computed from e(λ)): $\bar{\bar{\delta}}''_B(e(\lambda))$ and $\bar{\bar{\delta}}''_1(e(\lambda))$ are computed from the modified nearest-neighbor spectral radiation balance equations, weighted by the product $V(\lambda)J''_\lambda(\lambda,T_F)$, and integrated with respect to the wavelength over the visible spectrum (see the mathematical analysis section of this paper); these quantities do take into account variations in the spectral emissivity of the ceramic with wavelength.

2. Mathematical Analysis

2.1. Zonal Spectral Radiation Balance Equations

The spectral radiation balance equation for the j-th zone is

$$W_j'(\lambda) = e(\lambda)\left[J_\lambda(\lambda,T_j)/J_\lambda(\lambda,T_F)\right] + \sum_{k=B}^{N} F_{kj}W_k'(\lambda)r(\lambda). \qquad (1)$$

That is, the apparent spectral emissivity of the j-th zone at wavelength λ, referred to temperature T_F, is the corresponding spectral emissivity of the j-th zone plus the sum of the corresponding reflected apparent spectral emissivities of all zones (including the j-th zone itself).

2.2. Approximate Linearization of the Zonal Spectral Radiation Balance Equations for Small $\delta T_j'$

If the normalized base and wall-zone temperature drops (that is, the $\delta T_j'$) are very small (table 2 shows the base and wall-zone 1 normalized temperature drops to be of order of magnitude 10^{-6} for the NBS 1970 configuration), then $J_\lambda(\lambda,T_j)$ may be accurately represented by the first two terms of its Taylor series expansion about T_F. That is,

$$\left[J_\lambda(\lambda,T_j)/J_\lambda(\lambda,T_F)\right] = 1 - \delta T_j' N(\lambda,T_F). \qquad (2)$$

Now substitute for the left side of eq (2), as it appears in eq (1), the right side of eq (2). Further substitute

$$W_j'(\lambda) = 1 - \delta_j'(\lambda),\ W_k'(\lambda) = 1 - \delta_k'(\lambda),$$

into eq (1). The result is

$$1 - \delta_j'(\lambda) = e(\lambda)[1 - \delta T_j' N(\lambda,T_F)] + \sum_{k=B}^{N} F_{kj}[1 - \delta_k'(\lambda)]r(\lambda). \tag{3}$$

Now introduce the relation,

$$\sum_{k=B}^{N} F_{kj} = 1 - F_{Aj}. \tag{4}$$

That is, the surroundings viewed from zone j are completely covered by the base, the N wall zones, and the aperture.

Equation (3) may be transformed by the use of eq (4) into

$$\delta_j'(\lambda) = e(\lambda)\delta T_j' N(\lambda,T_F) + r(\lambda)[F_{Aj} + \sum_{k=B}^{N} F_{kj}\delta_k'(\lambda)]. \tag{5}$$

In other words, the spectral deficiency of the j-th zone is the sum of:

a. The spectral deficiency due to the temperature drop in the wall of the j-th zone.

b. The spectral deficiency due to the (diffuse) reflection of the aperture in the j-th zone.

c. The spectral deficiency due to the reflection of the spectral deficiencies of all the zones (including the j-th zone) in the j-th zone.

2.3. Nearest-Neighbor Approximation to the Zonal Spectral Radiation Balance Equations

It is clear from the viewfactor matrix (table 1) that the viewfactors between the zones fall off rapidly as the separation between the zones increases. Hence, if the zonal spectral deficiencies of the base and the adjacent wall zones are of the same order of magnitude (this assumption is justified below in sec. 2.4), it is justifiable to consider only the nearest-neighbors in approximating eq (5) for the zonal spectral radiation balance equations. Of course, the spectral deficiency of the aperture cannot be neglected, since it is nearly equal to 1 (that is, the effective temperature of the environment seen through the aperture is much less than the freezing point of platinum). Thus a good approximation to the zonal spectral radiation balance equation, eq (5), is

$$\delta'_j(\lambda)[1 - r(\lambda)F_{jj}] = e(\lambda)\delta T'_j N(\lambda,T_F) + r(\lambda)[F_{Aj} + F_{j-1,j}\delta'_{j-1}(\lambda) + F_{j+1,j}\delta'_{j+1}(\lambda)]. \qquad (6)$$

The coefficient, $[1 - r(\lambda)F_{jj}]$, is due to interreflections within the zone.

2.4. Zero-Order Approximation to the Zonal Spectral Radiation Balance Equations

To estimate the relative order of magnitude of the $\delta'_j(\lambda)$ and the $\delta T'_j$, the zero-order approximations, ${}_0\delta'_j(\lambda)$ and ${}_0\delta T'_j$, are useful. The zero-order approximations are defined by the zero-order zonal spectral radiation balance equation,

$${}_0\delta'_j(\lambda)[1 - r(\lambda)F_{jj}] = e(\lambda)\,{}_0\delta T'_j N(\lambda,T_F) + r(\lambda)F_{Aj}. \qquad (7)$$

The fractional error in ${}_0\delta'_j(\lambda)$, compared with $\delta'_j(\lambda)$, is roughly

$$E_j(\lambda) = [r^2(\lambda)(F_{A,j-1}F_{j-1,j} + F_{A,j+1}F_{j+1,j})]$$

$$X[r(\lambda)F_{Aj} + e(\lambda)\,{}_0\delta T'_j N(\lambda,T_F)]^{-1}. \qquad (8)$$

2.5. Zero-Order Power Balance Equations

The exact power balance equation for the j-th zone is

$$\delta T_j k/t = \int_0^\infty e(\lambda)J_\lambda(\lambda,T_j)d\lambda - \int_0^\infty e(\lambda)\sum_{k=B}^{N} F_{kj}W'_k(\lambda)J_\lambda(\lambda,T_F)d\lambda. \qquad (9)$$

Equation (9) simply states that the heat flow through the sighttube wall for a given zone (per unit interior area) is equal to the radiant power emitted by the zone minus the radiant power received by the zone. In other words, at equilibrium the net power input to the zone must be zero.

Next divide eq (9) through by σT_F^4, linearize, and substitute

$$W'_k(\lambda) = 1 - \delta'_k(\lambda), \quad \sum_{k=B}^{N} F_{kj} = 1 - F_{Aj},$$

into it to get

$$\delta T'_j(G^{-1} + 4\bar{e}') = \bar{e}F_{Aj} + \int_0^\infty e(\lambda)J'_\lambda(\lambda,T_F)\sum_{k=B}^{N} F_{kj}\delta'_k(\lambda)d\lambda. \qquad (10)$$

The nearest-neighbor approximation to eq (10) is

$$\delta T'_j(G^{-1} + 4\bar{e}') = \bar{e}F_{Aj} + \int_0^\infty e(\lambda)J'_\lambda(\lambda,T_F)[F_{j-1,j}\delta'_{j-1}(\lambda) +$$

$$F_{jj}\delta'_j(\lambda) + F_{j+1,j}\delta'_{j+1}(\lambda)]d\lambda. \qquad (11)$$

The zero-order approximation to eq (10) is

$${}_0\delta T'_j(G^{-1} + 4\bar{e}') = \bar{e}F_{Aj} + \int_0^\infty e(\lambda)J'_\lambda(\lambda,T_F)F_{jj}({}_0\delta'_j(\lambda))d\lambda. \qquad (12)$$

Now substitute into eq (12) the value of ${}_0\delta'_j(\lambda)$ from eq (7) to get

$${}_0\delta T'_j\left[G^{-1} + 4\bar{e}' - F_{jj}\int_0^\infty e^2(\lambda)N(\lambda,T_F)(1 - r(\lambda)F_{jj})^{-1}J'_\lambda(\lambda,T_F)d\lambda\right]$$

$$= F_{Aj}\int_0^\infty e(\lambda)(1 - r(\lambda)F_{jj})^{-1}J'_\lambda(\lambda,T_F)d\lambda. \qquad (13)$$

For the NBS 1970 configuration (composed of alumina), G^{-1} is 16.16 and $4\bar{e}'$ is 0.816; furthermore, the magnitude of the remaining term in the coefficient of ${}_0\delta T'_j$ in eq (13) is roughly $[4\bar{e}'^2_2F_{jj}/(1-\bar{r}F_{jj})]$. Thus for the base ($F_{jj} = 0.8$) the magnitude of this term is 0.447, while for the wall zones ($F_{jj} = 0.5858$) it is 0.227. It is clear, therefore, that the term G^{-1} in the coefficient of ${}_0\delta T'_j$ in eq (13) is much larger than the remaining terms. Hence a good approximation is

$${}_0\delta T'_j = GF_{Aj}\int_0^\infty e(\lambda)(1 - r(\lambda)F_{jj})^{-1}J'_\lambda(\lambda,T_F)d\lambda. \qquad (14)$$

Finally, numerical integration of the integral on the right side of eq (14) shows that there is less than 8% fractional error (for alumina and the viewfactors shown in table 1) in approximating $e(\lambda)$ and $r(\lambda)$ in the integrand by $\bar{e}$ and $\bar{r}$, respectively, to obtain

$${}_0\delta T'_j = GF_{Aj}\bar{e}/(1 - \bar{r}F_{jj}). \qquad (15)$$

2.6. Evaluation of the Fractional Error of the Zero-Order Approximations for $\delta T'_j$, $\bar{\delta}'_j$, and $\bar{\bar{\delta}}'_j$ (Compared with the Nearest-Neighbor Approximations)

Return to eq (7), the zero-order zonal spectral radiation balance equation, and substitute the value of ${}_0\delta T'_j$ given by eq (15) into eq (7); the result is

$$ {}_0\delta'_j(\lambda)[1 - r(\lambda)F_{jj}] = F_{Aj}[r(\lambda) + e(\lambda)N(\lambda,T_F)G\bar{e}(1-\bar{r}F_{jj})^{-1}]; \qquad (16) $$

eqs (15) and (16) point out the importance of the viewfactor of the aperture as viewed from zone j, F_{Aj}, in determining the magnitudes of ${}_0\delta T'_j$ and ${}_0\delta'_j(\lambda)$.

The approximate fractional error in ${}_0\delta'_j(\lambda)$, compared with the nearest-neighbor approximation to $\delta'_j(\lambda)$, eq (6), can now be evaluated by substituting into eq (8) for this fractional error the value of ${}_0\delta T'_j$ given by eq (15). Thus one derives

$$ \begin{aligned} E_j(\lambda) = {} & [\bar{r}^2(\lambda)/F_{Aj}][F_{A,j-1}F_{j-1,j} + F_{A,j+1}F_{j+1,j}] \\ & X[r(\lambda) + e(\lambda)N(\lambda,T_F)G\bar{e}(1-\bar{r}F_{jj})^{-1}]^{-1}. \end{aligned} \qquad (17) $$

The thermal average of $E_j(\lambda)$, $\bar{E}_j$, may be roughly computed from eq (17) as

$$ \begin{aligned} \bar{E}_j = {} & [\bar{r}^2/F_{Aj}][F_{A,j-1}F_{j-1,j} + F_{A,j+1}F_{j+1,j}] \\ & X[\bar{r} + 4\bar{e}^2G(1-\bar{r}F_{jj})^{-1}]^{-1}; \end{aligned} \qquad (18) $$

and the luminous average of $E_j(\lambda)$, $\bar{\bar{E}}_j$, may be similarly computed as

$$ \begin{aligned} \bar{\bar{E}}_j = {} & [\bar{\bar{r}}^2/F_{Aj}][F_{A,j-1}F_{j-1,j} + F_{A,j+1}F_{j+1,j}] \\ & X[\bar{\bar{r}} + \bar{\bar{N}}\bar{\bar{e}}G\bar{e}(1-\bar{r}F_{jj})^{-1}]^{-1}. \end{aligned} \qquad (19) $$

One may also estimate the fractional error in the zero-order normalized temperature drop of the j-th zone, ${}_0\delta T'_j$, compared with the nearest-neighbor approximation to the normalized temperature drop, by substituting ${}_0\delta'_{j-1}(\lambda)$ for $\delta'_{j-1}(\lambda)$, ${}_0\delta'_j(\lambda)$ for $\delta'_j(\lambda)$, and ${}_0\delta'_{j+1}(\lambda)$ for $\delta'_{j+1}(\lambda)$, in eq (11) for $\delta T'_j$, and proceeding as in the calculation of ${}_0\delta T'_j$ but retaining the terms involving ${}_0\delta'_{j-1}(\lambda)$ and ${}_0\delta'_{j+1}(\lambda)$. The result is

$$E_j({}_0\delta T'_j) = [\bar{r}/F_{Aj}][F_{A,j-1}F_{j-1,j}(1-\bar{r}F_{jj})(1-\bar{r}F_{j-1,j-1})^{-1} + F_{A,j+1}F_{j+1,j}(1-\bar{r}F_{jj})(1-\bar{r}F_{j+1,j+1})^{-1}]. \qquad (20)$$

Now insert into eqs (18), (19), and (20), for $\bar{E}_j$, $\bar{\bar{E}}_j$, and $E_j({}_0\delta T'_j)$, respectively, the values given in the list of terminology at the end of the introduction to this paper, for the NBS 1970 configuration for the standard of light; also insert the appropriate viewfactors from table 1. The resulting values for $\bar{E}_j$, $\bar{\bar{E}}_j$, and $E_j({}_0\delta T'_j)$, are shown in table 4. (Note that the effect of ceramic spectral emissivity variations with wavelength is not included in the fractional errors given in table 4, which account only for the omission of nearest-neighbor effects.) It is clear from table 4 that the zero-order approximation is good for the base, roughly satisfactory for wall-zone 2, and definitely not satisfactory for wall-zone 1.

2.7. Simultaneous Solution of the Spectral Radiation Balance Equations for the Base and Wall-Zone 1

It is clear from the results of table 4 that a valid approximation to the nearest-neighbor spectral radiation balance equation for wall-zone 1 is obtained by setting the spectral deficiency of wall-zone 2 equal to *zero*, since the spectral deficiency of wall-zone 2 as viewed from wall-zone 1 is less than 20% of the spectral deficiency of the *base* as viewed from wall-zone 1. In fact, an approximate calculation of the fractional error in the spectral radiation balance equation for wall-zone 1, due to omitting the spectral deficiency of wall-zone 2, indicates this error is less than 8% for both the thermal and the luminous deficiencies. This approximation is denoted the "modified nearest-neighbor" approximation.

In the modified nearest-neighbor approximation, the spectral radiation balance equations for the base and wall-zone 1 become,

$$\delta_B'(\lambda)[1 - r(\lambda)F_{BB}] = e(\lambda)\delta T_B' N(\lambda,T_F) + r(\lambda)[F_{AB} + F_{1B}\delta_1'(\lambda)], \tag{21}$$

and

$$\delta_1'(\lambda)[1 - r(\lambda)F_{11}] = e(\lambda)\delta T_1' N(\lambda,T_F) + r(\lambda)[F_{A1} + F_{B1}\delta_B'(\lambda)], \tag{22}$$

respectively.

The pair of spectral radiation balance equations above may be solved simultaneously for $\delta_B'(\lambda)$ and $\delta_1'(\lambda)$. The result is

$$\begin{aligned}\delta_B'(\lambda) = {} & [r(\lambda)(F_{11}F_{AB}e(\lambda)+F_{A1}F_{1B}r(\lambda)+2F_{AB}F_{B1}) + {} \\ & N(\lambda,T_F)e(\lambda)((F_{11}e(\lambda)+2F_{B1})\delta T_B' + F_{1B}r(\lambda)\delta T_1')]/[D(\lambda)]\end{aligned} \tag{23}$$

(where the identity, $1 - F_{11} = 2F_{B1}$, has been used), and

$$\begin{aligned}\delta_1'(\lambda) = {} & [r(\lambda)(F_{BB}F_{A1}e(\lambda)+F_{AB}F_{B1}r(\lambda)+F_{A1}(1-F_{BB})) + {} \\ & N(\lambda,T_F)e(\lambda)((F_{BB}e(\lambda)+1-F_{BB})\delta T_1' + F_{B1}r(\lambda)\delta T_B')]/[D(\lambda)],\end{aligned} \tag{24}$$

where

$$D(\lambda) = \left[e^2(\lambda)(F_{BB}F_{11}-F_{B1}F_{1B}) + e(\lambda)(F_{BB}+F_{11}-2F_{BB}F_{11}+2F_{B1}F_{1B}) + \right.$$
$$\left. ((1-F_{BB})(1-F_{11}) - F_{B1}F_{1B})\right]. \qquad (25)$$

2.8. Modified Nearest-Neighbor Approximation to the Power Balance Equations for the Base and Wall-Zone 1

By reasoning similar to that used in section 2.7 to justify omitting the contribution of the spectral deficiency of wall-zone 2 to the spectral radiation balance equation for wall-zone 1, the spectral deficiency of wall-zone 2 can also be omitted from the power balance equation for wall-zone 1. Thus the modified nearest-neighbor approximation for wall-zone 1 leads to the power balance equations,

$$\delta T_B'(G^{-1} + 4\bar{e}') = \bar{e}F_{AB} + \int_0^\infty e(\lambda)J_\lambda'(\lambda,T_F)\left[F_{BB}\delta_B'(\lambda) + F_{1B}\delta_1'(\lambda)\right]d\lambda, \qquad (26)$$

and

$$\delta T_1'(G^{-1} + 4\bar{e}') = \bar{e}F_{A1} + \int_0^\infty e(\lambda)J_\lambda'(\lambda,T_F)\left[F_{B1}\ {}_B'(\lambda) + F_{11}\ {}_1'(\lambda)\right]d\lambda, \qquad (27)$$

for the base and wall-zone 1, respectively.

Furthermore, by reasoning similar to that used in section 2.5 to justify omitting terms of order G^2 from the zero-order power balance equations for the base and wall-zone 1, terms of order G^2 may also be omitted from the corresponding modified nearest-neighbor power balance equations. Thus, after substituting the values of $\delta_B'(\lambda)$ and $\delta_1'(\lambda)$ given by eqs (23) and (24) into eqs (26) and (27), one derives the approximate modified nearest-neighbor power balance equations for the base and wall-zone 1,

$$[\delta T_B''/G] = \bar{e} + \int_0^\infty e(\lambda)r(\lambda)J_\lambda'(\lambda,T_F)\frac{[0.8279-0.4343r(\lambda)]}{[1-1.3858r(\lambda)+0.4343r^2(\lambda)]}d\lambda, \tag{28}$$

and

$$[\delta T_1''/G] = 0.1349\bar{e} + \int_0^\infty e(\lambda)r(\lambda)J_\lambda'(\lambda,T_F)\frac{[0.2861-0.0586r(\lambda)]}{[1-1.3858r(\lambda)+0.4343r^2(\lambda)]}d\lambda, \tag{29}$$

respectively.

Numerical integration of eqs (28) and (29) leads to the values given in table 2. The values for $\sin\theta = 1$ refer to the NBS 1970 configuration with a flat base (the actual base is conical, but the flat base values are included for comparison). Note that the normalized base and wall-zone 1 temperature drops have been divided by the viewfactor of the aperture as viewed from the base, F_{AB} (approximately equal to $\sin\theta/\bar{L}^2$). For the conical base ($\sin\theta = 0.2$), F_{AB} is approximately 1/8000; for the flat base, F_{AB} is approximately 1/1600. That is, for the conical base,

$$\delta T_B'(e(\lambda)) = 0.759\text{x}10^{-4}G, \quad \delta T_1'(e(\lambda)) = 0.278\text{x}10^{-4}G;$$

while for the flat base,

$$\delta T_B'(e(\lambda)) = 1.74\text{x}10^{-4}G, \quad \delta T_1'(e(\lambda)) = 0.529\text{x}10^{-4}G.$$

2.9. Luminous Deficiencies for the Base and Wall-Zone 1, Computed: a., from the Luminous Average of the Spectral Deficiencies; b., from the Spectral Deficiencies at the Crova Wavelength

a. Luminous Deficiencies Computed from the Luminous Average of the Spectral Deficiencies

By definition, the luminous deficiencies of the base and wall-zone 1 are

$$\bar{\bar{\delta}}'_B = \int_0^\infty \delta'(\lambda)V(\lambda)J''_\lambda(\lambda,T_F)d\lambda, \tag{30}$$

and

$$\bar{\bar{\delta}}'_1 = \int_0^\infty \delta'(\lambda)V(\lambda)J''_\lambda(\lambda,T_F)d\lambda, \tag{31}$$

respectively.

Since $\delta T''_B(e(\lambda))$, $\delta T''_B(\bar{e})$, $\delta T''_1(e(\lambda))$, and $\delta T''_1(\bar{e})$, have now been calculated, eqs (23) and (24) for $\delta'_B(\lambda)$ and $\delta'_1(\lambda)$ are therefore completely determined as functions of wavelength. Hence $\bar{\bar{\delta}}'_B$ and $\bar{\bar{\delta}}'_1$ may be obtained from eqs (30) and (31) by numerical integration. Because $\delta T'_B(e(\lambda))$ and $\delta T'_1(e(\lambda))$ take account of the spectral variations in $e(\lambda)$, these quantities - rather than $\delta T'_B(\bar{e})$ and $\delta T'_1(\bar{e})$ - are substituted into eqs (23) and (24) for $\delta'_B(\lambda)$ and $\delta'_1(\lambda)$. $\delta'_B(\lambda)$ and $\delta'_1(\lambda)$ are then substituted into eqs (30) and (31) to obtain $\bar{\bar{\delta}}'_B$ and $\bar{\bar{\delta}}'_1$. Since $\bar{\bar{\delta}}'_B$ and $\bar{\bar{\delta}}'_1$ computed in this manner do take account of the spectral variations in $e(\lambda)$, they are denoted $\bar{\bar{\delta}}'_B(e(\lambda))$ and $\bar{\bar{\delta}}'_1(e(\lambda))$. The results of these calculations are shown in table 3.

b. Luminous Deficiencies Computed from the Spectral Deficiencies at the Crova Wavelength

As mentioned in the introduction to this paper (sec. 1.1.c), the Sanders-NRC computer program uses the average thermal emissivity of the sighttube ceramic, $\bar{e}$, to compute the temperatures of the base and the wall zones. An approximation to this procedure is simply to replace $e(\lambda)$ by $\bar{e}$ in the power balance equations for the base and wall-zone 1 (eqs (26) and (27), respectively), and proceed as before to compute $\delta T''_B(\bar{e})$ and $\delta T''_1(\bar{e})$. (These quantities are denoted $\delta T''_B(\bar{e})$ and $\delta T''_1(\bar{e})$ to indicate that $\bar{e}$ instead of $e(\lambda)$ has been used in their calculation; $\delta T''_B(\bar{e})$ and $\delta T''_1(\bar{e})$ do not take account of the spectral variations in $e(\lambda)$.) Table 2 presents $\delta T''_B(e(\lambda))$, $\delta T''_B(\bar{e})$, $\delta T''_1(e(\lambda))$, and $\delta T''_1(\bar{e})$.

After computing the temperatures of the base and the wall zones, the Sanders-NRC program uses these values in conjunction with the average luminous emissivity of the sighttube ceramic, $\bar{\bar{e}}$, to compute the spectral emissivities of the base and the wall zones at the Crova wavelength, λ_C. As explained in the introduction, the Sanders-NRC computer program assumes that the apparent spectral emissivities at the Crova wavelength are equal to the corresponding apparent luminous emissivities.

An approximation to this procedure is to replace $e(\lambda)$ by $\bar{\bar{e}}$, $r(\lambda)$ by $\bar{\bar{r}}$, $N(\lambda,T_F)$ by $\bar{\bar{N}}$, and $\delta T'_B$ and $\delta T'_1$ by $\delta T'_B(\bar{e})$ and $\delta T'_1(\bar{e})$, respectively, in eqs (23) and (24) for $\delta'_B(\lambda)$ and $\delta'_1(\lambda)$. The resulting expressions for $\delta'_B(\lambda)$ and $\delta'_1(\lambda)$ - when substituted into eqs (30) and (31) - yield values for $\bar{\bar{\delta}}'_B$ and $\bar{\bar{\delta}}'_1$. Since $\bar{\bar{\delta}}'_B$ and $\bar{\bar{\delta}}'_1$ computed in this manner do not take account of the spectral variations in $e(\lambda)$, they are denoted $\bar{\bar{\delta}}'_B(\bar{\bar{e}})$ and $\bar{\bar{\delta}}'_1(\bar{\bar{e}})$. The results of these calculations are also shown in table 3.

3. Conclusions

It is concluded from the results presented in table 3, that the effect of the variations in the spectral emissivity of the sighttube ceramic with wavelength, on the accuracy of the apparent luminous emissivities of the base and the adjacent wall of the sighttube, as computed by the Sanders-NRC computer program, is *negligible* for the NBS 1970 design for the standard of light, composed of alumina, compared with the other uncertainties and the precision of measurement. (An approximate calculation of the errors involved in the use of the modified nearest-neighbor approximation, which was used to derive the results given in tables 2 and 3, was obtained by evaluating the most important terms omitted from this approximation. The calculation shows that the errors produced by the use of the modified nearest-neighbor approximation do *not* significantly affect the results of table 2 or table 3.)

Inspection of the spectral emissivity variations with wavelength of alumina, magnesia, and thoria (given in refs. 11 and 13), shows that alumina has the greatest variation with wavelength of these 3 ceramics. This is true for both the visible range and the range containing most of the radiated energy of a 2045 K blackbody (roughly 0.5 to 12 micrometers for 99% of the radiated energy). Thus the differences shown in table 3 between $\bar{\bar{\delta}}''_B(e(\lambda))$ and $\bar{\bar{\delta}}''_B(\bar{\bar{e}})$, and between $\bar{\bar{\delta}}''_1(e(\lambda))$ and $\bar{\bar{\delta}}''_1(\bar{\bar{e}})$, for the NBS 1970 configuration made of alumina, would probably be *less* if the calculations in this paper were carried through for magnesia or thoria.

The results of table 3 also demonstrate that the sighttube with the conical base produces smaller luminous deficiencies than the corresponding sighttube with a *flat* base (the cylindrical portions having equal length-to-diameter ratios). This is shown by multiplying the values given in table 3 for $\bar{\bar{\delta}}''_B(e(\lambda))$ and $\bar{\bar{\delta}}''_1(e(\lambda))$ by the appropriate aperture-base viewfactors (1/8000 for the conical base, 1/1600 for the flat base) to derive: for the conical base,

$$\bar{\bar{\delta}}'_B(e(\lambda)) = 2.44\text{x}10^{-4},\ \bar{\bar{\delta}}'_1(e(\lambda)) = 0.88\text{x}10^{-4};$$

while for the flat base,

$$\bar{\bar{\delta}}'_B(e(\lambda)) = 5.50\text{x}10^{-4},\ \bar{\bar{\delta}}'_1(e(\lambda)) = 1.68\text{x}10^{-4}.$$

Thus the luminous deficiencies for the conical base geometry are roughly 1/2 the corresponding values for the flat base. Therefore, the above luminous deficiencies are not proportional to $\sin\theta$, since the value of $\sin\theta$ is 1/5 for the conical base and 1 for the flat base.

The effect of the conical base, in reducing the luminous deficiency of the base of the sighttube of the NBS 1970 design for the standard of light, can be estimated from the zero-order approximation. Referring to eq (16) in section 2.6, and approximating the zero-order value of $\bar{\bar{\delta}}'_B$ by

$$ {}_0\bar{\bar{\delta}}'_B = F_{AB}[\bar{\bar{r}} + \bar{\bar{N}}\bar{\bar{e}}G\bar{e}(1-\bar{\bar{r}}F_{BB})^{-1}]/[1 - \bar{\bar{r}}F_{BB}], \qquad (32)$$

one derives

$$[{}_0\bar{\bar{\delta}}'_B(\theta)/{}_0\bar{\bar{\delta}}'_B(\pi/2)] = [\sin\theta(\bar{\bar{r}} + \bar{\bar{N}}\bar{\bar{e}}G\bar{e}(\bar{e}+\bar{\bar{r}}\sin\theta)^{-1})]$$

$$\text{X}[(\bar{\bar{e}} + \bar{\bar{r}}\sin\theta)(\bar{\bar{r}} + \bar{\bar{N}}\bar{\bar{e}}G\bar{e})]^{-1}. \qquad (33)$$

When the values appropriate to the NBS 1970 design made of alumina (given in the list of terminology at the end of the introduction) are substituted into eq (33), it is found that

$${}_0\bar{\bar{\delta}}'_B(\theta)/{}_0\bar{\bar{\delta}}'_B(\pi/2) = 0.48;$$

this is close to the more accurate value (from col. 2 of table 3),

$$\bar{\bar{\delta}}'_B(\theta)/\bar{\bar{\delta}}'_B(\pi/2) = 0.45.$$

It can also be seen from eq (33) that if $\bar{\bar{r}}\sin\theta \gg \bar{\bar{e}}$ and $\bar{\bar{r}} \gg \bar{\bar{N}}\bar{\bar{e}}G\bar{e}$, then approximately,

$${}_0\bar{\bar{\delta}}'_B(\theta)/{}_0\bar{\bar{\delta}}'_B(\pi/2) = 1/\bar{\bar{r}}.$$

In other words, the effect of a high average luminous reflectivity of the ceramic composing the sighttube is to eliminate (for $\sin\theta$ not too small) the advantage of the conical base which holds for low values of ceramic luminous reflectivity. For if $\bar{\bar{r}}\sin\theta \ll \bar{\bar{e}}$, but still $\bar{\bar{r}} \gg \bar{\bar{N}}\bar{\bar{e}}G\bar{e}$, then approximately,

$${}_0\bar{\bar{\delta}}'_B(\theta)/{}_0\bar{\bar{\delta}}'_B(\pi/2) = \sin\theta/\bar{\bar{e}}.$$

The basic conclusion of this report is that the error introduced into the values of the apparent luminous emissivities of the base and the adjacent wall of the NBS 1970 design for the standard of light, made of alumina, as computed by the Sanders-NRC computer program, due to neglect of ceramic spectral emissivity variations with wavelength, is of the order of 5×10^{-6} (see sec. 1.7). This is negligible, compared with the uncertainties in the computed values produced by uncertainties in the thermal and radiative properties of alumina, and compared with the uncertainties which present techniques achieve in photometric intercomparisons.

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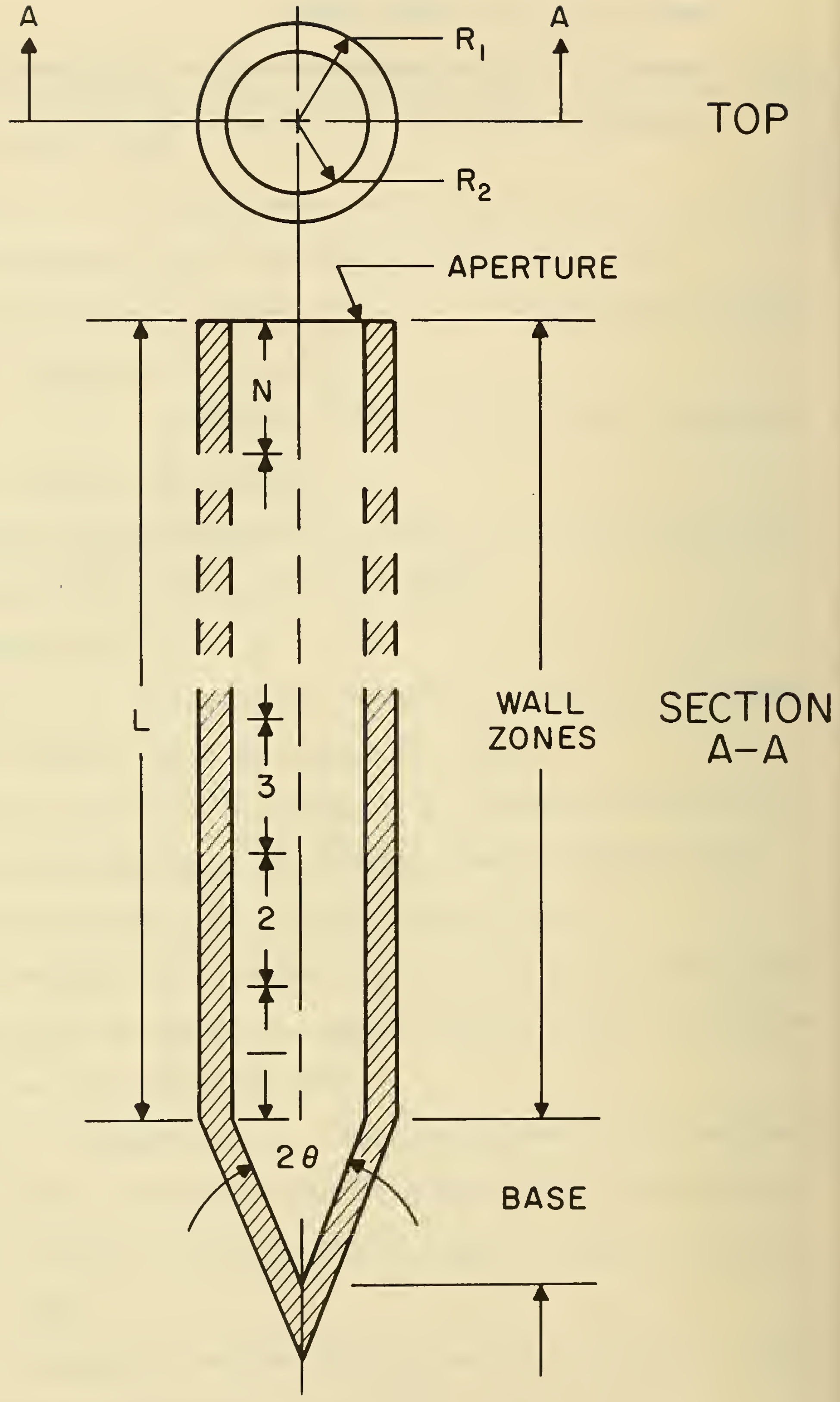

Figure 1. Standard of light; sighttube geometry.

Figure 2. Simplified flow chart for the approximate method of calculating the luminous deficiencies of the base and wall-zone 1 - of the NBS 1970 design for the standard of light - that accounts for variations in the spectral emissivity of the ceramic with wavelength (see fig. 1; see list of terminology for definitions of symbols).

1. Data Input

L, R_1, R_2, θ, k, T_F, $e(\lambda)$, $N(\lambda,T_F)$, $J'_\lambda(\lambda,T_F)$, $J''_\lambda(\lambda,T_F)$, $V(\lambda)$.

2. Zone Partition

Subdivide interior of sighttube into N wall zones (plus the base); compute viewfactor matrix (F_{jk}).

3. Spectral Radiation Balance Equations for Base and Wall-Zone 1

a. Set up the spectral radiation balance equations for the base and wall-zone 1.

b. Make modified nearest-neighbor approximation.

c. Solve spectral radiation balance equations for base and wall-zone 1 for $\delta''_B(\lambda)$ and $\delta''_1(\lambda)$.

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4. Power Balance Equations for Base and Wall-Zone 1

a. Set up power balance equations for the base and wall-zone 1.

b. Make modified nearest-neighbor approximation.

c. Discard terms of order G^2.

d. Integrate the power balance equations with respect to wavelength from zero to infinity (nominally) to obtain $\delta T''_B(e(\lambda))$ and $\delta T''_1(e(\lambda))$.

5. Luminous Deficiencies of Base and Wall-Zone 1

a. Substitute $\delta T''_B(e(\lambda))$ and $\delta T''_1(e(\lambda))$ into the equations for $\delta''_B(\lambda)$ and $\delta''_1(\lambda)$.

b. Integrate $\delta''_B(\lambda)V(\lambda)J''_\lambda(\lambda,T_F)$ and $\delta''_1(\lambda)V(\lambda)J''_\lambda(\lambda,T_F)$ with respect to wavelength over the visible spectrum to obtain $\bar{\bar{\delta}}''_B(e(\lambda))$ and $\bar{\bar{\delta}}''_1(e(\lambda))$.

Figure 3. Simplified flow chart for the approximate method of calculating the luminous deficiencies of the base and wall-zone 1 - of the NBS 1970 design for the standard of light - that neglects variations in the spectral emissivity of the ceramic with wavelength (see fig. 1; see list of terminology for definitions of symbols).

1. Data Input

L, R_1, R_2, θ, k, T_F, $\bar{e}$, $\bar{\bar{e}}$, $\bar{\bar{N}}$.

2. Zone Partition

Subdivide interior of sighttube into N wall zones (plus the base); compute viewfactor matrix (F_{jk}).

3. Spectral Radiation Balance Equations for Base and Wall-Zone 1

a. Set up the spectral radiation balance equations for the base and wall-zone 1; $e(\lambda)$ is replaced by $\bar{e}$.

b. Make modified nearest-neighbor approximation.

c. Solve the spectral radiation balance equations for the base and wall-zone 1 for $\delta''_B(\lambda)$ and $\delta''_1(\lambda)$.

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4. Power Balance Equations for the Base and Wall-Zone 1

a. Set up power balance equations for the base and wall-zone 1.

b. Make modified nearest-neighbor approximation.

c. Discard terms of order G^2.

d. Solve the power balance equations for the base and wall-zone 1 for $\delta T''_B(\bar{e})$ and $\delta T''_1(\bar{e})$.

5. Luminous Deficiencies of Base and Wall-Zone 1

a. Substitute $\delta T''_B(\bar{e})$ and $\delta T''_1(\bar{e})$ into the equations for $\delta''_B(\lambda)$ and $\delta''_1(\lambda)$; $e(\lambda)$ is replaced by $\bar{\bar{e}}$.

b. Evaluate $\delta''_B(\lambda)$ and $\delta''_1(\lambda)$ at the Crova wavelength (λ_C) to obtain $\bar{\bar{\delta}}''_B(\bar{e})$ and $\bar{\bar{\delta}}''_1(\bar{e})$.

Figure 4. Simplified flow chart for the Sanders-NRC computer program for calculating the luminous deficiencies of the base and wall zones of the standard of light; this program neglects variations in the spectral emissivity of the ceramic with wavelength (see fig. 1; see list of terminology for definitions of symbols).

1. Data Input

L, R_1', R_2, θ, k, T_F, $\bar{e}$, $\bar{\bar{e}}$, λ_C.

2. Zone Partition

Subdivide interior of sighttube into N wall zones (plus the base); compute viewfactor matrix (F_{jk}).

3. Thermal Radiation Balance Equations and Power Balance Equations

a. Set up N+1 spectral radiation balance equations for the base and N wall zones; $e(\lambda)$ is replaced by $\bar{e}$.

b. Set up N+1 apparent thermal radiation balance equations for the base and the N wall zones by integrating the corresponding spectral radiation balance equations with respect to wavelength from zero to infinity (nominally).

c. Set up N+1 power balance equations for the base and the N wall zones.

d. Solve the system of 2N+2 equations (b. plus c.) simultaneously, by iteration, for the N+1 interior temperatures and the N+1 apparent thermal emissivities of the base and the N wall zones.

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4. Apparent Luminous Emissivities of Base and Wall Zones
   a. Substitute the N+1 interior temperatures of the base and the N wall zones into the corresponding spectral radiation balance equations; $e(\lambda)$ is replaced by $\bar{\bar{e}}$.
   b. Solve the system of N+1 equations (a.) simultaneously, by iteration, at the Crova wavelength (λ_C) for the N+1 apparent luminous emissivities of the base and the N wall zones.

Table 1. Viewfactor matrix for cylindrical sighttube with conical base; zone length equals one diameter (see fig. 1).

Viewed Zone	Viewing Zone Base	 Wall 1	 Wall 2	 Wall 3	 Wall N	 Aperture
Base	$1 - \sin\theta$	0.2071	0.0290	0.0073	$^{a}(\bar{L}-1)^{-3}$	$(\bar{L})^{-2}$
Wall 1	$\sin\theta(0.8284)$	0.5858	0.1781	0.0216	$6(\bar{L}-2)^{-4}$	$4(\bar{L}-1)^{-3}$
Wall 2	$\sin\theta(0.1158)$	0.1781	0.5858	0.1781	$6(\bar{L}-4)^{-4}$	$4(\bar{L}-3)^{-3}$
Wall 3	$\sin\theta(0.0294)$	0.0216	0.1781	0.5858	$6(\bar{L}-6)^{-4}$	$4(\bar{L}-5)^{-3}$
Wall N	$4\sin\theta(\bar{L}-1)^{-3}$	$6(\bar{L}-2)^{-4}$	$6(\bar{L}-4)^{-4}$	$6(\bar{L}-6)^{-4}$	0.5858	0.8284
Aperture	$\sin\theta(\bar{L})^{-2}$	$(\bar{L}-1)^{-3}$	$(\bar{L}-3)^{-3}$	$(\bar{L}-5)^{-3}$	0.2071	0.0000

[a] Viewfactors with $\bar{L}$ ($\bar{L}=L/R_2$) are approximate; $\bar{L}$ is equal to 40 for the simplified model of the NBS 1970 design for the standard of light.

Table 2. Approximate normalized temperature drops (divided by the aperture-base viewfactor, F_{AB}) for the base and wall-zone 1 (see fig. 1) of the NBS 1970 design for the standard of light (made of alumina), computed by two different methods: method 1 accounts for variations in the spectral emissivity of the ceramic with wavelength; method 2 neglects variations in the spectral emissivity of the ceramic with wavelength (see list of terminology for definitions of symbols). Fractional errors in method 2, assuming method 1 to be correct.

	Normalized Base Temperature Drop, Divided by F_{AB}			Normalized Wall-Zone 1 Temperature Drop, Divided by F_{AB}		
$\sin\theta$	(Method 1) $\delta T''_B(e(\lambda))$	(Fractional Error in Method 2)	(Method 2) $\delta T''_B(\bar{e})$	(Method 1) $\delta T''_1(e(\lambda))$	(Fractional Error in Method 2)	(Method 2) $\delta T''_1(\bar{e})$
0.2	0.6073G	0.122	0.6815G	0.2225G	0.146	0.2549G
1.0	0.2783G	0.022	0.2844G	0.0846G	0.123	0.0950G

Table 3. Approximate luminous deficiencies (divided by the aperture-base viewfactor, F_{AB}) for the base and wall-zone 1 (see fig. 1) of the NBS 1970 design for the standard of light (made of alumina), computed by two different methods: method 1 accounts for variations in the spectral emissivity of the ceramic with wavelength; method 2 neglects variations in the spectral emissivity of the ceramic with wavelength (see list of terminology for definitions of symbols). Fractional errors in method 2, assuming method 1 to be correct.

	Base Luminous Deficiency, Divided by F_{AB}			Wall-Zone 1 Luminous Deficiency, Divided by F_{AB}		
sinθ	(Method 1) $\bar{\bar{\delta}}''_B(e(\lambda))$	(Fractional Error in Method 2)	(Method 2) $\bar{\bar{\delta}}''_B(\bar{\bar{e}})$	(Method 1) $\bar{\bar{\delta}}''_1(e(\lambda))$	(Fractional Error in Method 2)	(Method 2) $\bar{\bar{\delta}}''_1(\bar{\bar{e}})$
0.2	1.953	0.025	1.905	0.702	0.044	0.671
1.0	0.880	0.010	0.872	0.269	0.028	0.261

Table 4. Zero-order approximations to the normalized temperature drops and the thermal and luminous deficiencies for the base and wall-zones 1 and 2 (see fig. 1) of the NBS 1970 design for the standard of light (made of alumina). Fractional errors in the zero-order approximations, assuming the nearest-neighbor approximations to be correct (see list of terminology for definitions of symbols).

Zone	Zero-Order Normalized Temperature Drop ${}_0\delta T'_j$	Zero-Order Thermal Deficiency ${}_0\bar{\delta}'_j$	Zero-Order Luminous Deficiency ${}_0\bar{\bar{\delta}}'_j$	Fractional Error in Zero-Order Normalized Temperature Drop $E_j({}_0\delta T'_j)$	Fractional Error in Zero-Order Thermal Deficiency $\bar{E}_j$	Fractional Error in Zero-Order Luminous Deficiency $\bar{\bar{E}}_j$
Base	[a]0.046(-4)	2.68(-4)	2.13(-4)	0.014	0.017	0.012
Wall 1	0.004(-4)	0.25(-4)	0.21(-4)	1.60	1.32	0.98
Wall 2	0.005(-4)	0.29(-4)	0.24(-4)	0.28	0.27	0.20

[a] The symbol "(-4)" indicates the preceding number is to be multiplied by 10^{-4}.

U. S. GOVERNMENT PRINTING OFFICE : 1971—481-325/61

FORM NBS-114A (1-71)

U.S. DEPT. OF COMM. **BIBLIOGRAPHIC DATA SHEET**	1. PUBLICATION OR REPORT NO. NBS-TN-595	2. Gov't Accession No.	3. Recipient's Accession No.

4. TITLE AND SUBTITLE Effect of Ceramic Spectral Emissivity Variations on the Computed Luminous Emissivity of the NBS Standard of Light	5. Publication Date **September 1971**
	6. Performing Organization Code
7. AUTHOR(S) William B. Fussell, Physicist	8. Performing Organization
9. PERFORMING ORGANIZATION NAME AND ADDRESS NATIONAL BUREAU OF STANDARDS DEPARTMENT OF COMMERCE WASHINGTON, D.C. 20234	10. Project/Task/Work Unit No. 2211125
	11. Contract/Grant No.
12. Sponsoring Organization Name and Address **Same as No. 9.**	13. Type of Report & Period Covered not applicable
	14. Sponsoring Agency Code

15. SUPPLEMENTARY NOTES

16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

A simplified model of the National Bureau of Standards (NBS) 1970 design for the standard of light (platinum point blackbody) is introduced. This model is used to calculate the apparent luminous emissivity of the base of the sighttube of the standard of light, and of the wall near the base, by two different methods. The first method includes the effect of variations with wavelength in the spectral emissivity of the ceramic composing the sighttube; the second method does not include the effect of ceramic spectral emissivity variations. The results of the two methods are compared and their difference (about $6x10^{-6}$) is found to be negligible for the NBS 1970 design when the sighttube is made of alumina, compared with other uncertainties and the precision of measurement.

A formula is derived for estimating the apparent luminous emissivity of a standard of light, given: a., the length-to-diameter ratio of the sighttube; b., the half-angle of the conical base; c., the wall thickness of the sighttube; d., the thermal conductivity of the sighttube ceramic; e., the average thermal and luminous emissivities of the sighttube ceramic.

17. KEY WORDS (Alphabetical order, separated by semicolons)

Ceramic; light; photometry; radiometry; spectral emissivity; standard.

18. AVAILABILITY STATEMENT ☒ UNLIMITED. ☐ FOR OFFICIAL DISTRIBUTION. DO NOT RELEASE TO NTIS.	19. SECURITY CLASS (THIS REPORT) UNCLASSIFIED	21. NO. OF PAGES 58
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